Lepton Transmutation in e^+e^- Collision

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Abstract

It is shown that the lepton flavour-violating reactions: $e^+e^- \longrightarrow e^\pm\mu^\mp, e^\pm\tau^\mp, \mu^\pm\tau^\mp$, all occur by virtue of the rotating lepton mass matrix which, even in the Standard Model, is a consequence of the renormalization group equation if neutrinos oscillate, and may be driven by further forces in schemes beyond the Standard Model. Following a procedure developed earlier, we show that the full differential cross section is unambiguously calculable for any rotation mechanism once given the overall normalization which, however, depends on how the mass matrix rotation is driven. Calculations done with the Dualized Standard Model at 10.58 GeV, the operating energy of BaBar and Belle, give cross sections for $e^+e^- \longrightarrow e^\pm\tau^\mp$ and $\mu^\pm\tau^\mp$ in the multi-fb range, which should be detectable with the present sensitivity of these experiments. Confirmation of these predictions not only checks renormalization theory but offers deep new insight into the origin of fermion generations.

1 Introduction

One consequence of neutrino oscillation, which has been strongly indicated if not already confirmed by recent experiments [1, 2], is that the lepton mass matrix will rotate with changing energy scales even in the traditional Standard Model. The reason is that, like other quantities such as the running coupling constant, the mass matrix in quantum field theory satisfies certain renormalization group equations [3] which dictate how the mass matrix should vary as the scale changes, and given neutrino oscillations, i.e. nontrivial mixing between the up (neutrinos) and down (charged leptons) states, these equations in the Standard Model imply that the mass matrix must change its orientation in generation space (rotate) as the scale varies [4].

Going beyond the traditional Standard Model, one is likely to encounter further mechanisms for driving the mass matrix rotation. That the mass matrix rotates means that its eigenstates at some given scale will in general no longer be eigenstates at some other scales, or that generation states under changing scales will rotate into one another, and this has important implications for the generation problem. First, the fact alone that generation states can be rotated into one another already means that they are not independent entities as once perceived but just different manifestations of the same object like the different colours of a quark [5, 6]. It further suggests that generations are related by some continuous "horizontal" symmetry [7], and if this symmetry is gauged, as all known continuous symmetries seem to be, then new forces will arise in association with it which can contribute to the mass matrix rotation also. In considering the effects of mass matrix rotation therefore, it would seem prudent to allow for other possible mechanisms driving the rotation beyond that induced by the nontrivial mixing matrix as described in the preceding paragraph. Indeed, we think that there are even empirical reasons why it may be attractive to entertain the possibility of the mass matrix rotation giving rise to fermion mixing [8], rather than the other way round. In any case, our main interest in this paper is the calculation of rotation effects in general terms, independent of the mechanism driving the rotation, although we shall of course also consider some specific examples.

That the lepton mass matrix rotates with changing scales means that at an arbitrary energy it will not in general be diagonal in the flavour states e, μ, τ of the charged leptons which are defined as eigenstates of the mass matrix at the scales of their respective masses. Reaction amplitudes will thus in general have nondiagonal elements linking leptons of different flavours leading to a new category of lepton-flavour violations ("transmutations") different from those due, for example, to flavour-changing neutral currents (FCNC). Such transmutational flavour-violating effects occur by virtue of mass matrix rotation even when there are no explicit flavour-violating coupling in the action, and can be sizeable even when FCNC effects are small. Indeed, it has already been shown in earlier papers [5, 9, 6], to which the reader is referred also for more extended versions of the preceding arguments, that in certain cases, depending on the mechanism driving the rotation, the predicted effects can be within the sensitivity range of present experiments. The observation of these effects will not only confirm the basic tenets of the renormalization procedure in quantum field theory which requires the mass matrix to rotate given nontrivial mixing, but also supply us with new valuable insight into the fundamental question of the origin of fermion generations. The size of the effect, if any, and its variation with energy will give indications on the nature of this symmetry and how the rotation of the mass matrix is driven, the knowledge of which will in turn shed light on the origin of generations itself, a basic question in particle physics that has already been with us for many years.

In the present paper, we choose to investigate transmutational effects in Bhabha scattering, namely the reactions:

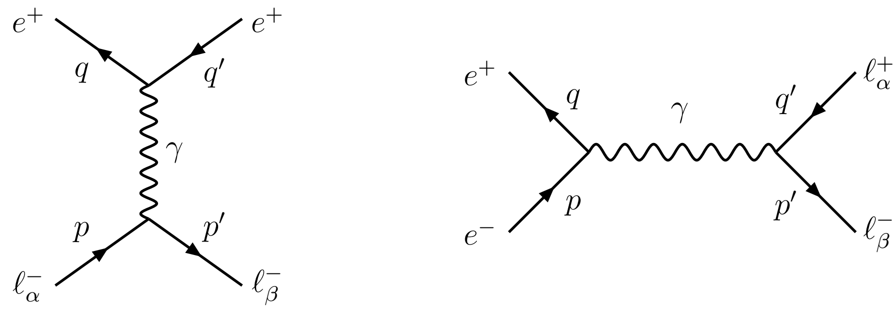
$$e^+e^- \longrightarrow e^+\tau^-, \tau^+e^-; \tag{1}$$

$$e^+e^- \longrightarrow e^+\mu^-, \mu^+e^-; \tag{2}$$

$$e^+e^- \longrightarrow \mu^+\tau^-, \tau^+\mu^-. \tag{3}$$

The obvious practical reason for doing so is that there are several high intensity machines in operation, such as BEPC, CESR (Cleo), PEP II (BaBar), and KEK II (Belle), which appear capable of observing these effects, besides LEP, which though sadly just turned off, has left still masses of data which could be useful for the same purpose. Indeed, apart from the leptonic decays of certain mesons [6] which, being single particle effects, have only limited capability for testing the theory, Bhabha scattering is probably the first example one would naturally consider for studying lepton transmutation, and the only reason we have not already done this before photo-transmutation [9] was that in the beginning with imperfect understanding, we were under the erroneous impression that only amplitudes with internal fermion propagators can give transmutations.

According to our present understanding, however, lepton transmutation can proceed in e^+e^- collisions by, for example, the processes represented by the Feynman diagrams in Figure 1. At the energy at which an experiment is performed, the amplitudes for these processes, being dependent on the lepton masses, are diagonal in the eigenstates $j = 1, 2, 3$ of the mass matrix at that scale but not in general, by the reasoning above, diagonal in the flavour states e, μ, τ. And this fact alone, by the same reasoning, is enough to give lepton transmutation as a result.



(a) (b)

Figure 1: Feynman diagrams for the transmutation amplitude.

The transmutation reactions (1), (2), (3) can of course occur via other processes, such as higher order photon-exchange diagrams or Z_0-exchange, but the effects from these for the energy range of present interest is small so that in what follows, we shall be concerned with only the diagrams in Figure 1. What we shall show is that, by following the procedure suggested in [9] originally for photo-transmutation which we find can be adapted to the present case, one can calculate explicitly the cross sections for the 3 transmutational reactions (1), (2), and (3), given the rotating mass matrix from any rotation scheme.

In particular, for the so-called Dualized Standard Model (DSM) scheme we ourselves advocate, which purports to explain fermion generations and their mixing [10] and gives explicitly the rotation matrix, the transmutations (1) and (3) are both found to be well within range of detection by such sensitive experiments as BaBar [11], Belle [12], and Cleo [13]. Indeed, a detailed study of the effects at the $\Upsilon(4S)$ mass at which these experiments are run reveals that BaBar, for example, can expect to see as many as a few hundred events for each of the two reactions in the data they have already collected from last year's run of 20 fb^{-1}, assuming 100 percent detection efficiency. In other words, with the existing data, one should be able already to effect an unambiguous test of the DSM's (parameter-free) predictions.

2 The Amplitude (a)

Consider first the one-photon exchange diagram of Figure 1(a), which will be seen to give the dominant contribution to the two reactions (1) and (2). It does not contribute to the reaction (3), which can proceed by one-photon exchange in e^+e^- collision only when both e^+ and e^- transmute, but this will be so far down in magnitude as to be negligible for present consideration. Then according to the procedure suggested in [9], at any given energy $\sqrt{s}$, the transmutation amplitude for the reaction :

$$e^+\ell^-_\alpha \longrightarrow e^+\ell^-_\beta, \tag{4}$$

is given by a rotation in generation space, thus:

$$\mathcal{M}^{(a)} = \sum_j S^\dagger_{\beta j}\mathcal{M}^{(a)}_j S_{\alpha j}, \tag{5}$$

from the diagonal amplitudes $\mathcal{M}^{(a)}_j$ for the reaction:

$$e^+\ell^-_j \longrightarrow e^+\ell^-_j \tag{6}$$

for the mass eigenstate j at the scale $\sqrt{s}$ with eigenvalue m_j, where $S_{\alpha j} = \langle j|\alpha\rangle$ is the rotation matrix in generation space which relates the triad of lepton flavour states $\alpha = e, \mu, \tau$ to the eigentriad j at the scale $\sqrt{s}$. Explicitly, for the one-photon exchange diagram of Figure 2(a), we have:

$$(\mathcal{M}^{(a)}_j)^{r'r}_{s's} = -ie^2[\bar{u}_{s'}(p'_j)\gamma^\mu u_s(p_j)]\frac{1}{(p'_j - p_j)^2}[\bar{v}_r(q)\gamma_\mu v_{r'}(q')], \tag{7}$$

where s and s' denote the spins of the incoming and outgoing lepton ℓ^- and r and r' those of the antileptons ℓ^+.

To actually evaluate (7), we have yet to specify the values of the momenta p_j and p'_j entering there. The point is that the amplitude for the two-body reaction (6) is of course a function of only two variables, which we may take to be the standard Mandelstam variables s and t, so that all components of the various momenta appearing in (7) must be expressible in terms of

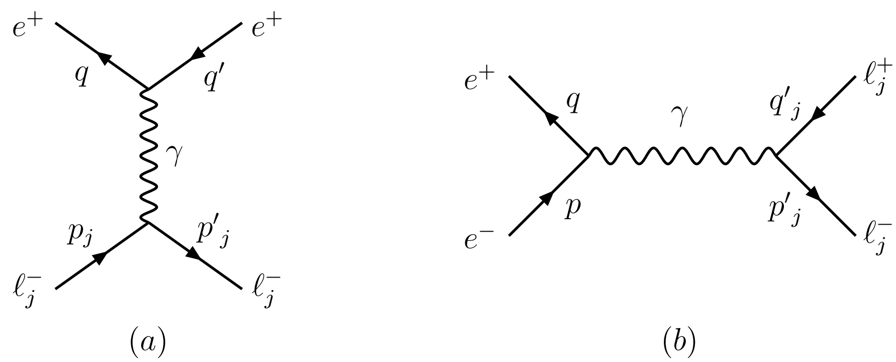


Figure 2: Feynman diagram for the diagonal amplitudes.

them. The reasoning required to arrive at these expressions is not entirely trivial, but following the considerations given in [9] which apply as well to the present case with but minor modifications, we obtain the following relationships between the different momenta to be used later for deriving the required expressions:

$$\begin{aligned} p_j &= a_j q + b_j q' + c_j p_i \\ p'_j &= a_j q' + b_j q + c_j p'_i, \end{aligned} \tag{8}$$

where

$$\begin{aligned} c_j &= \sqrt{\frac{(s - m_j^2 + m^2)^2 + s t_0}{(s - m^2)^2 + s t}}, \\ b_j &= \frac{c_j(s + m^2) - (s - m_j^2 + m^2)}{t_0}, \\ a_j &= \frac{c_j(s + m^2 + t_0) - (s - m_j^2 + m^2 + t_0)}{t_0}, \end{aligned} \tag{9}$$

with $t_0 = t - 4m^2$, m the positron mass, and m_i put equal to zero for reasons to be made apparent.

In this paper, we shall be interested mainly in unpolarized cross sections which means that we shall need to evaluate sums of the absolute values squared of the amplitudes (5) over all the spins s, r, s', r'. These spin-sums, as was explained in [9], are not so readily performed as usual by the standard method of taking traces of γ-matrices because of the crossed terms between channels labelled by different j's obtained in squaring (5). We shall therefore follow the tactics adopted in [9] of explicitly performing the spin-sums in a specific Lorentz frame with a specific representation of the γ-matrices. As in [9], we choose to work in the cm frame of the channel $i = 3$ which in our convention denotes the mass eigenstate with the lowest mass m_3, which, being in all cases considered at most of the order of the electron mass and therefore negligible, is put equal to zero. This gives:

$$\begin{aligned} q^\mu &= (E, 0, 0, \omega), \\ q'^\mu &= (E, 0, \omega \sin\theta'_3, \omega\cos\theta'_3), \\ p_3^\mu &= (\omega, 0, 0, -\omega), \\ p_3'^\mu &= (\omega, 0, -\omega\sin\theta'_3, -\omega\cos\theta'_3), \\ p_j^\mu &= (E_j, 0, -\omega_j \sin\theta_j, -\omega_j\cos\theta_j), \\ p_j'^\mu &= (E_j, 0, -\omega_j\sin\theta'_j, -\omega_j\cos\theta'_j), \end{aligned} \tag{10}$$

with

$$\begin{aligned} E &= \frac{s+m^2}{2\sqrt{s}}, \\ \omega &= \frac{s-m^2}{2\sqrt{s}}, \\ \cos\theta'_3 &= 1 + \frac{2st}{(s-m^2)^2}. \end{aligned} \tag{11}$$

Further, from (8)–(10), one obtains

$$\begin{aligned} E_j = \frac{1}{2\sqrt{s}t_0}\Big\{ & 2\sqrt{[(s-m^2)^2+st][(s-m_j^2+m^2)^2+st_0]} \\ & -(s+m^2)[2(s-m^2-m_j^2)+t]\Big\}, \end{aligned} \tag{12}$$

with

$$\omega_j = \sqrt{E_j^2 - m_j^2}, \tag{13}$$

and

$$\sin\theta_j = \frac{-\sqrt{-st}}{2st_0\omega\omega_j}\Big\{(s+m^2)\sqrt{(s-m_j^2+m^2)^2+st_0}$$

$$
\begin{aligned}
&\quad - (s - m_j^2 + m^2)\sqrt{(s-m^2)^2 + st}\Big\},\\
\sin\theta'_j \quad &= \quad \frac{-\sqrt{-st}}{2st_0\omega\omega_j}\Big\{(s+m^2)\sqrt{(s-m_j^2+m^2)^2+st_0}\\
&\quad - (s - m_j^2 + m^2 + t_0)\sqrt{(s-m^2)^2+st}\Big\},
\end{aligned} \tag{14}
$$

with

$$
\theta'_3 = \theta_j + \theta'_j. \tag{15}
$$

These formulae (11)–(14), together with the formulae in the last paragraph, all reduce to the corresponding formulae derived in [9] for photo-transmutation if we put the mass m of the e^+ in (6) equal to zero, which will indeed be a very good approximation in most applications. We have kept the dependence on m explicit only for the sake of generality in case the formulae are to be applied in future to other circumstances, such as lepton transmutations in $\mu^+\mu^-$ collisions.

We choose again for γ-matrices the Pauli–Dirac representation:

$$
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};\ \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}. \tag{16}
$$

The spins of the incoming and outgoing leptons j we quantize along the direction p_3 and p'_3 respectively, while the spins of the e^+, whether incoming or outgoing, we quantize along its direction of motion. With these specifications, the wave functions of the leptons j are given by:

$$
\begin{aligned}
u_+(p_j) \quad &= \quad \frac{1}{\sqrt{2(E_j+m_j)}}\begin{pmatrix} 0 \\ E_j + m_j \\ 0 \\ \omega_j e^{-i\theta_j} \end{pmatrix},\\
u_-(p_j) \quad &= \quad \frac{1}{\sqrt{2(E_j+m_j)}}\begin{pmatrix} E_j + m_j \\ 0 \\ -\omega_j e^{-i\theta_j} \\ 0 \end{pmatrix},
\end{aligned} \tag{17}
$$

and

$$
u_+(p'_j) \quad = \quad \frac{1}{2\sqrt{2(E_j+m_j)}}\begin{pmatrix} (E_j+m_j)(1-e^{-i\theta'_3}) \\ (E_j+m_j)(1+e^{-i\theta'_3}) \\ \omega_j(e^{i\theta_j} - e^{-i\theta'_j}) \\ \omega_j(e^{i\theta_j} + e^{-i\theta'_j}) \end{pmatrix},
$$

$$u_-(p'_j) \;=\; \frac{1}{2\sqrt{2(E_j+m_j)}}\begin{pmatrix}(E_j+m_j)(1+e^{-i\theta'_3})\\(E_j+m_j)(1-e^{-i\theta'_3})\\-\omega_j(e^{i\theta_j}+e^{-i\theta'_j})\\-\omega_j(e^{i\theta_j}-e^{-i\theta'_j})\end{pmatrix}; \qquad (18)$$

while for the e^+, we have the wave functions:

$$\begin{aligned} v_+(q) &= \frac{1}{\sqrt{2(E+m)}}\begin{pmatrix}0\\-\omega\\0\\E+m\end{pmatrix}, \\ v_-(q) &= \frac{1}{\sqrt{2(E+m)}}\begin{pmatrix}\omega\\0\\E+m\\0\end{pmatrix}, \end{aligned} \qquad (19)$$

and:

$$\begin{aligned} v_+(q') &= \frac{1}{2\sqrt{2(E+m)}}\begin{pmatrix}-\omega(1-e^{-i\theta'_3})\\-\omega(1+e^{-i\theta'_3})\\(E+m)(1-e^{-i\theta'_3})\\(E+m)(1+e^{-i\theta'_3})\end{pmatrix}, \\ v_-(q') &= \frac{1}{2\sqrt{2(E+m)}}\begin{pmatrix}\omega(1+e^{-i\theta'_3})\\\omega(1-e^{-i\theta'_3})\\(E+m)(1+e^{-i\theta'_3})\\(E+m)(1-e^{-i\theta'_3})\end{pmatrix}. \end{aligned} \qquad (20)$$

With the wave functions in (17)–(20), it is straightforward to evaluate the diagonal amplitudes in (7). We obtain the following:

$$\begin{aligned} (\mathcal{M}_j^{(a)})^{++}_{++} &= (\mathcal{M}_j^{(a)})^{--}_{--} \\ &= -\frac{ie^2}{t}\Big\{\frac{E}{4}(1+\cos\theta'_3)[(E_j+m_j)+(E_j-m_j)e^{-2i\theta_j}] \\ &\quad +\omega\omega_j(1-\cos\theta'_3)e^{-i\theta_j}+\frac{\omega\omega_j}{2}(1+\cos\theta'_3)e^{-i\theta_j}\Big\} \\ (\mathcal{M}_j^{(a)})^{--}_{++} &= (\mathcal{M}_j^{(a)})^{++}_{--} \\ &= -\frac{ie^2}{t}(1+\cos\theta'_3)\Big\{\frac{E}{4}[(E_j+m_j)+(E_j-m_j)e^{-2i\theta_j}] \end{aligned}$$

$$
\begin{aligned}
& \quad + \frac{\omega\omega_j}{2} e^{-i\theta_j} \Big\} \\
(\mathcal{M}_j^{(a)})^{+-}_{+-} &= (\mathcal{M}_j^{(a)})^{-+}_{-+} = (\mathcal{M}_j^{(a)})^{-+}_{+-} = (\mathcal{M}_j^{(a)})^{+-}_{-+} \\
&= -\frac{ie^2}{t}\frac{m}{4}(1-\cos\theta_3')\Big\{(E_j+m_j)-(E_j-m_j)e^{-2i\theta_j}\Big\} \\
(\mathcal{M}_j^{(a)})^{+-}_{++} &= (\mathcal{M}_j^{(a)})^{-+}_{++} = (\mathcal{M}_j^{(a)})^{+-}_{--} = (\mathcal{M}_j^{(a)})^{-+}_{--} \\
&= \frac{e^2}{t}\frac{m}{4}\sin\theta_3'\Big\{(E_j+m_j)+(E_j-m_j)e^{-2i\theta_j}\Big\} \\
(\mathcal{M}_j^{(a)})^{++}_{+-} &= (\mathcal{M}_j^{(a)})^{++}_{-+} = (\mathcal{M}_j^{(a)})^{--}_{+-} = (\mathcal{M}_j^{(a)})^{--}_{-+} \\
&= -\frac{e^2}{t}\frac{E}{4}\sin\theta_3'\Big\{(E_j+m_j)-(E_j-m_j)e^{-2i\theta_j}\Big\}, \qquad (21)
\end{aligned}
$$

where subscripts denote the spins of the leptons j and superscripts the spins of the e^+, with the right index pertaining to the incoming and the left index to the outgoing particle. Using the formulae derived earlier in (11)–(14), these amplitudes can then be expressed in terms of the Mandelstam invariants s and t as desired.

3 The Amplitude (b)

Turning next to the diagram of Figure 1(b) which contributes to all three reactions (1)–(3), we proceed in a similar manner. The transmutational amplitude for the reaction:

$$e^+e^- \longrightarrow \ell^+_\alpha \ell^-_\beta, \qquad (22)$$

is given by a rotation in generation space, thus:

$$\mathcal{M}^{(b)} = \sum_j S_{\alpha j} S^\dagger_{\beta j} \mathcal{M}_j^{(b)}, \qquad (23)$$

from the diagonal amplitudes $\mathcal{M}_j^{(b)}$ depicted in Figure 2(b) for the reaction:

$$e^+e^- \longrightarrow \ell^+_j \ell^-_j \qquad (24)$$

where

$$(\mathcal{M}_j^{(b)})^{r'r}_{s's} = ie^2[\bar{u}_{s'}(p_j')\gamma^\mu v_{r'}(q_j')]\frac{1}{(p+q)^2}[\bar{v}_r(q)\gamma_\mu u_s(p)]. \qquad (25)$$

We work now in the cm of the incoming e^+ and e^- system with

$$\begin{aligned}
q^\mu &= (E, 0, 0, \omega), \\
p^\mu &= (E, 0, 0, -\omega), \\
p'^\mu_j &= (E_j, 0, -\omega_j \sin\theta'_j, -\omega_j \cos\theta'_j), \\
q'^\mu_j &= (E_j, 0, \omega_j \sin\theta'_j, \omega_j \cos\theta'_j),
\end{aligned} \tag{26}$$

and

$$\begin{aligned}
E &= E_j = \sqrt{s}/2, \\
\omega &= \sqrt{E^2 - m^2}; \quad \omega_j = \sqrt{E^2 - m_j^2}, \\
\cos\theta'_3 &= 1 + \frac{t}{2E^2}, \\
\cos\theta'_j &= \frac{t - m^2 - m_j^2 + 2E^2}{2\omega\omega_j}.
\end{aligned} \tag{27}$$

Further, with the spins of the outgoing particles ℓ_j^+, ℓ_j^- quantized along q'_3 and p'_3 respectively, and those of the incoming e^+, e^- along their directions of motion q and p respectively, the wave functions are given by:

$$\begin{aligned}
u_+(p) &= \frac{1}{\sqrt{2(E+m)}} \begin{pmatrix} 0 \\ E+m \\ 0 \\ \omega \end{pmatrix}, \\
u_-(p) &= \frac{1}{\sqrt{2(E+m)}} \begin{pmatrix} E+m \\ 0 \\ -\omega \\ 0 \end{pmatrix}, \\
v_+(q) &= \frac{1}{\sqrt{2(E+m)}} \begin{pmatrix} 0 \\ -\omega \\ 0 \\ E+m \end{pmatrix}, \\
v_-(q) &= \frac{1}{\sqrt{2(E+m)}} \begin{pmatrix} \omega \\ 0 \\ E+m \\ 0 \end{pmatrix},
\end{aligned} \tag{28}$$

and

$$
\begin{aligned}
u_+(p'_j) &= \frac{1}{2\sqrt{2(E_j+m_j)}} \begin{pmatrix} (E_j+m_j)(1-e^{-i\theta'_j}) \\ (E_j+m_j)(1+e^{-i\theta'_j}) \\ \omega_j(1-e^{-i\theta'_j}) \\ \omega_j(1+e^{-i\theta'_j}) \end{pmatrix}, \\
u_-(p'_j) &= \frac{1}{2\sqrt{2(E_j+m_j)}} \begin{pmatrix} (E_j+m_j)(1+e^{-i\theta'_j}) \\ (E_j+m_j)(1-e^{-i\theta'_j}) \\ -\omega_j(1+e^{-i\theta'_j}) \\ -\omega_j(1-e^{-i\theta'_j}) \end{pmatrix}, \\
v_+(q'_j) &= \frac{1}{2\sqrt{2(E_j+m_j)}} \begin{pmatrix} -\omega_j(1-e^{-i\theta'_j}) \\ -\omega_j(1+e^{-i\theta'_j}) \\ (E_j+m_j)(1-e^{-i\theta'_j}) \\ (E_j+m_j)(1+e^{-i\theta'_j}) \end{pmatrix}, \\
v_-(q'_j) &= \frac{1}{2\sqrt{2(E_j+m_j)}} \begin{pmatrix} \omega_j(1+e^{-i\theta'_j}) \\ \omega_j(1-e^{-i\theta'_j}) \\ (E_j+m_j)(1+e^{-i\theta'_j}) \\ (E_j+m_j)(1-e^{-i\theta'_j}) \end{pmatrix}.
\end{aligned} \tag{29}
$$

Hence, one obtains the amplitudes:

$$
\begin{aligned}
(\mathcal{M}_j^{(b)})^{++}_{++} &= (\mathcal{M}_j^{(b)})^{--}_{--} = -\frac{ie^2}{s} m m_j \cos\theta'_j, \\
(\mathcal{M}_j^{(b)})^{--}_{++} &= (\mathcal{M}_j^{(b)})^{++}_{--} = -\frac{ie^2}{s} E E_j (1+\cos\theta'_j), \\
(\mathcal{M}_j^{(b)})^{+-}_{+-} &= (\mathcal{M}_j^{(b)})^{-+}_{-+} = \frac{ie^2}{s} m m_j \cos\theta'_j, \\
(\mathcal{M}_j^{(b)})^{+-}_{++} &= (\mathcal{M}_j^{(b)})^{-+}_{--} = \frac{e^2}{s} E m_j \sin\theta'_j, \\
(\mathcal{M}_j^{(b)})^{++}_{+-} &= (\mathcal{M}_j^{(b)})^{--}_{-+} = -\frac{e^2}{s} E m_j \sin\theta'_j, \\
(\mathcal{M}_j^{(b)})^{+-}_{--} &= (\mathcal{M}_j^{(b)})^{-+}_{++} = \frac{e^2}{s} m E_j \sin\theta'_j, \\
(\mathcal{M}_j^{(b)})^{-+}_{+-} &= (\mathcal{M}_j^{(b)})^{+-}_{-+} = -\frac{ie^2}{s} E E_j (1-\cos\theta'_j) \\
(\mathcal{M}_j^{(b)})^{++}_{-+} &= (\mathcal{M}_j^{(b)})^{--}_{+-} = -\frac{e^2}{s} m E_j \sin\theta'_j,
\end{aligned} \tag{30}
$$

which again can all be expressed in terms of the Mandelstam invariants s and t by means of the formulae in (27).

4 Spin-summed Differential Cross Sections

Substituting the diagonal amplitudes in (21) and (30) into respectively the formulae (5) and (23) and adding the two contributions, one obtains the spin-amplitudes for the actual transmutation reaction (1), (2) and (3). Hence, taking the absolute values squared of these amplitudes and summing over all spins, one obtains the spin-summed differential cross sections as desired:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\frac{1}{s}\frac{\omega'}{\omega}\frac{1}{4}\sum_{r,r';s,s'}|(\mathcal{M})^{r'r}_{s's}|^2 \times 0.3894 \text{ mb/sr}, \tag{31}$$

for energies measured in GeV, where ω and ω' are respectively the cm momenta of the actual incoming and outgoing particles in the transmutation reaction. Although this is in principle straightforward, a few practical observations are in order.

First, in performing the sum in (5) and (23) over the diagonal states j, it is convenient to make use of the unitary property of the rotation matrix $S_{\alpha j}$ to write, for $\alpha \neq \beta$:

$$\sum_j S_{\alpha j} S^\dagger_{\beta j} \mathcal{M}_j = S_{\alpha 1} S^\dagger_{\beta 1}[\mathcal{M}_1 - \mathcal{M}_3], \tag{32}$$

which, as explained in [9], is a good approximation whenever the mass eigenvalues m_j are hierarchical and avoids the need to know the rotation matrix to unreasonably high accuracy. Besides, as we shall see, it gives us a clearer picture of how transmutation cross sections behave as functions of the Mandelstam invariants s and t.

Second, as can be seen in (32), the transmutation amplitude being proportional to $\mathcal{M}_1 - \mathcal{M}_3$ with the two amplitudes differing just by the mass values, i.e. whether m_1 or m_3, the cross section for transmutation is at most of order m_1^2/s and decreases rapidly with increasing energy. For high s, therefore, $\mathcal{M}_1$ and $\mathcal{M}_3$ will largely cancel leading potentially to inaccuracy in a direct calculation with the formula (32). Indeed, this was exactly what we found in our actual calculations, especially in the the amplitude (a) where other large cancellations occur in the exact formulae. This computational difficulty can be avoided just by expanding the amplitudes to order m_1^2/s giving:

$$
\begin{aligned}
(\mathcal{M}^{(a)})^{++}_{++} &= (\mathcal{M}^{(a)})^{--}_{--} \\
&= \frac{ie^2}{t}\left\{\frac{m_1^2}{2s}(s-t) - im_1^2\sqrt{\frac{-t}{s+t}}\right\}, \\
(\mathcal{M}^{(a)})^{--}_{++} &= (\mathcal{M}^{(a)})^{++}_{--} \\
&= \frac{ie^2}{t}(s+t)\left\{\frac{m_1^2}{2s} - i\frac{m_1^2}{s}\sqrt{\frac{-t}{s+t}}\right\}, \\
(\mathcal{M}^{(a)})^{++}_{+-} &= (\mathcal{M}^{(a)})^{++}_{-+} = (\mathcal{M}^{(a)})^{--}_{+-} = (\mathcal{M}^{(a)})^{--}_{-+} \\
&= \frac{e^2}{2}\sqrt{\frac{s+t}{-t}}\left\{\frac{m_1}{\sqrt{s}} - i\frac{m_1^2}{s}\sqrt{\frac{-t}{s+t}}\right\}, \qquad (33)
\end{aligned}
$$

and all other components zero, where we have also neglected terms of the order of the electron mass m. This approximation is already very good by $\sqrt{s}$ of order 10 GeV, at least near the forward direction where the amplitudes are large, and becomes eventually necessary above this energy for computations without double precision. In Figure 3 is shown the spin-summed differential cross sections for the reaction $e^+e^- \longrightarrow e^+\tau^-$ at $\sqrt{s} = 10$ GeV, 100 GeV calculated with the rotation matrix element $S_{\alpha 1}$ of the DSM scheme taken from ref. [5, 14]. The curve at 10 GeV is calculated with the exact formulae (21) which is seen to be almost indistinguishable from the crosses calculated with the approximate formulae (33). The curve at 100 GeV is calculated with (33) where the exact formulae is found to have problems with accuracy in application.

Third, we note that the two sets of diagonal amplitudes (21) and (30) were each calculated in a particular Lorentz frame, namely for the diagram (a) in the cm frame of the $e^+\ell_3^-$ system and for the diagram (b) in the cm frame of the incoming e^+e^- system. Although the amplitudes were all converted in the end into functions of the invariants s and t, the directions of spin quantization are still frame-dependent. Hence, strictly speaking, the two frames for (a) and (b) being different, the two respective spin-amplitudes could not be added in the manner that we have done above. However, the electron mass m is so small compared to the energies we are interested in that this difference in frame is entirely negligible for practical purposes. Were the present formalism to be adapted in future to say $\mu^+\mu^-$ collisions, then this would be a point to be borne in mind.

With these points clarified, we have not encountered any more practical

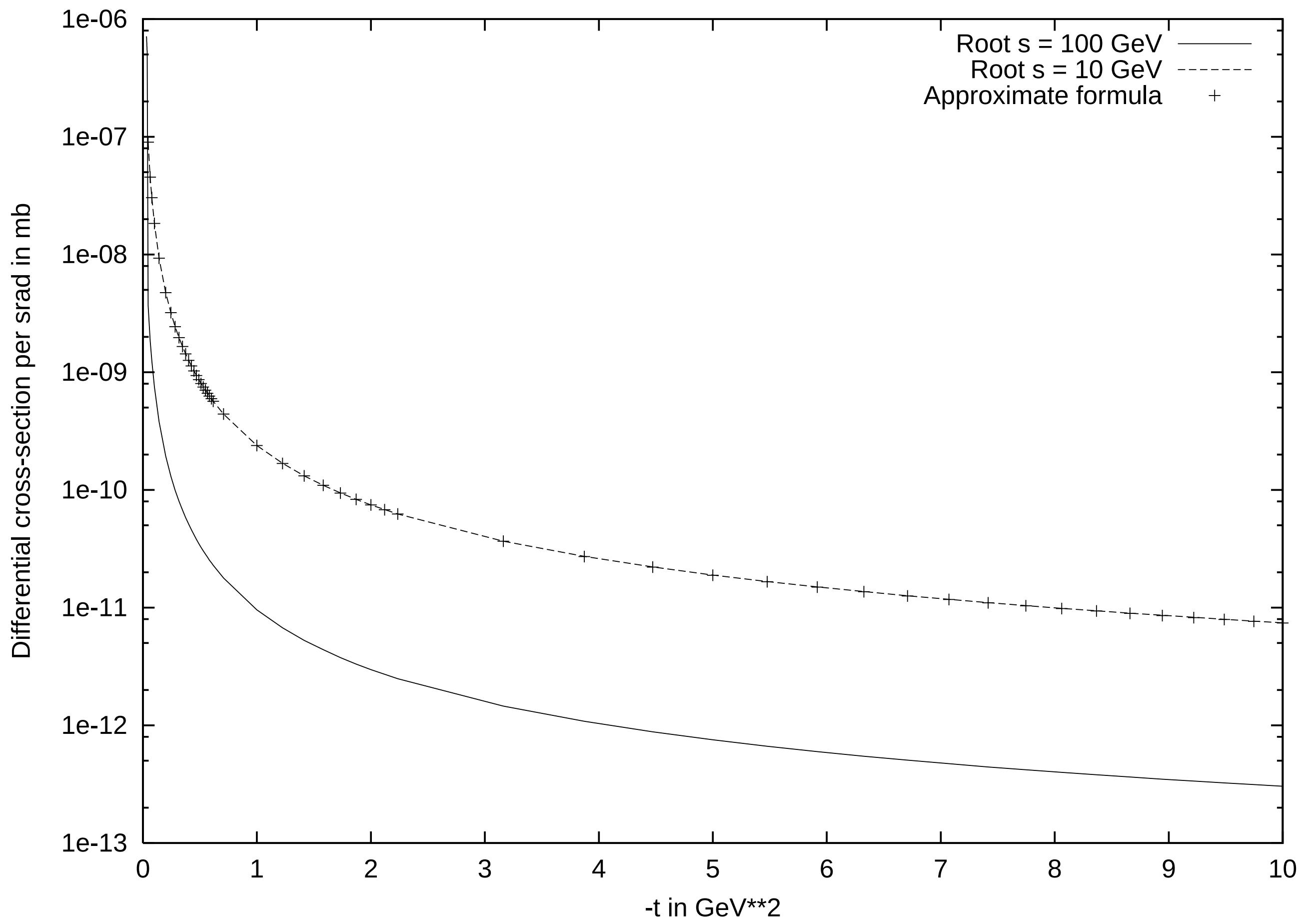


Figure 3: Spin-summed differential cross sections for the reaction $e^+e^- \longrightarrow e^+\tau^-$ at $\sqrt{s} = 10$ GeV, 100 GeV. For details see text.

difficulties in computing the cross sections of the three transmutation reactions (1)–(3). Rather than presenting our results in a wide range of s and t, which could be confusing, we shall instead first give here a description of the general features, and then in the next section a detailed report on the result at $\sqrt{s} = 10.58$ GeV, namely at the $\Upsilon(4S)$ where BaBar [11], Belle [12], and Cleo [13] have already collected a massive amount of data, in principle ready to be confronted with our predictions.

Consider first the reactions (1) and (2) which receive contributions from both diagrams (a) and (b), and are very similar except for the difference in normalization due to the different values of the rotation matrix elements $S_{\alpha 1}$. As in ordinary Bhabha scattering, the amplitude (a) is dominated by the pole at $t = 0$ which gives the cross section a sharp forward peak, as can be seen in the examples of Figure 3. Except at large scattering angles where t is of order s, this peak overshadows the contribution from the (b) diagram. However, the forward peak for the transmutation reactions (1) and (2) is nowhere near as sharp as for ordinary Bhabha scattering, as can be seen in Figure 4. The reason for this difference is seen in (33), where one notices that the normally dominant spin non-flip amplitudes with $1/t$ behaviour are of order m_1^2/s, while the spin flip with a weaker $1/\sqrt{-t}$ behaviour are of order $m_1/\sqrt{s}$. The same formulae (33) explains also the sharp decline of the cross section with increasing energy as well as its change in t-dependence as the spin flip terms become ever more dominant, both of which effects can be seen in Figure 3 by comparing the curves at $\sqrt{s} = 10$ and 100 GeV.

The other reaction (3) receives contributions only from the (b) diagram which has no peak in the forward direction. It is distinguished from the same diagram in ordinary Bhabha scattering by the fact that, like the (a) transmutation amplitude, it is also dominated by the spin flip terms at high energy. Without the sharp singular peak in the forward direction, it gives, in contrast to reactions (1) and (2), a finite total cross section, the rough energy dependence of which is shown in Figure 5, where one sees that, as in photo-transmutation [9], the cross section rises shortly after threshold to a peak and then declines as $\sqrt{s}$ increases.

We note that all the actual numbers shown in Figures 3–5 were calculated with rotation matrix elements $S_{\alpha j}$ of the DSM scheme taken from ref. [14]. Since $S_{\alpha j}$ appear only in the normalization of the cross section, the latter's dependence on t, and qualitatively also on s, is not affected by the choice. For more discussion on the rotation matrix $S_{\alpha j}$ in different schemes, the reader is referred to [5, 6], and to the following section for details in the case at hand.

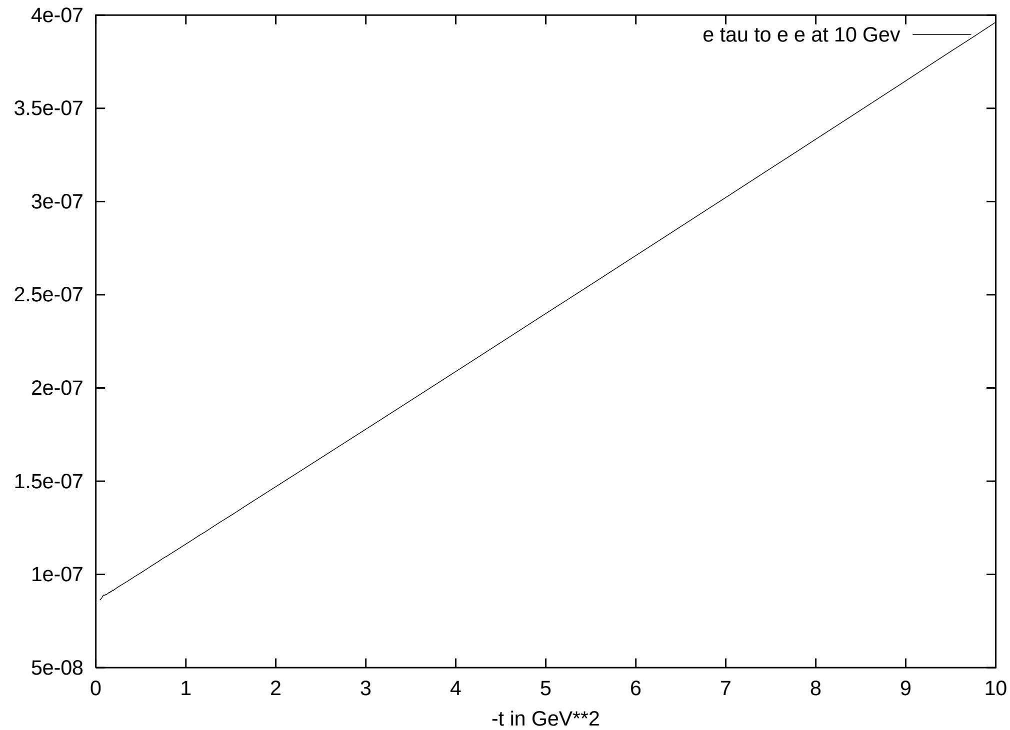


Figure 4: The ratio at 10 GeV of the cross section of reaction $e^+e^- \longrightarrow e^+\tau^-$ over that of ordinary Bhabha scattering $e^+e^- \longrightarrow e^+e^-$ as a function of t.

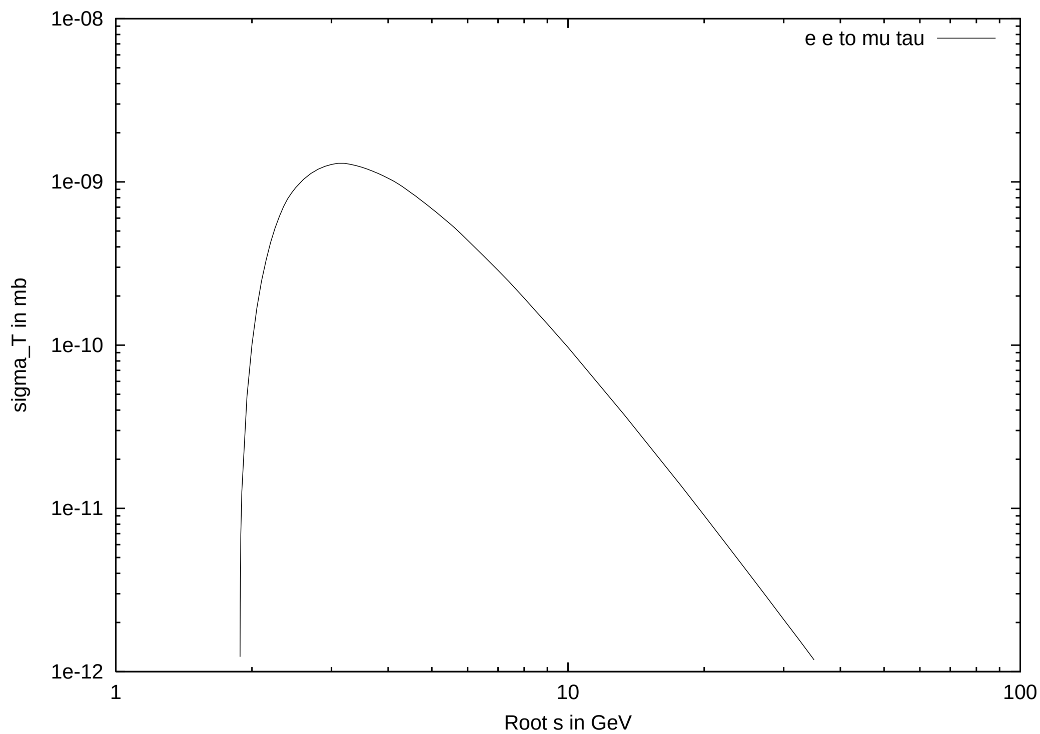


Figure 5: Cross section for the reaction $e^+e^- \longrightarrow \mu^+\tau^-$ integrated over all scattering angles as a function of $\sqrt{s}$.

5 Transmutation Cross Sections at $\sqrt{s} = 10.58$ GeV

The reason for selecting this particular energy corresponding to the mass of the $\Upsilon(4S)$ for detailed analysis is that two experiments of ultra-high sensitivity, namely BaBar and Belle, are running and have each collected already some 20 fb^{-1} of luminosity, with much more expected in the near future [11, 12]. Although these experiments were designed originally to look for other rare effects like CP-violation from B decay, their data could conveniently be used also to search for the transmutation reactions of interest to us here.

The spin-summed differential cross section for the reaction (1) at 10.58 GeV as calculated from Figure 1 with the rotation matrix element $S_{\alpha 1}$ of the DSM scheme from [5, 14] is shown in Figure 6 over the whole angular range. There is in principle also a contribution from the transmutational decay of the $\Upsilon(4S)$ resonance but this is seen to be negligible [6] in comparison. As $S_{\alpha 1}$ appears in the amplitude (32) only in the normalization factor, the choice of a different scheme for the rotating mass matrix will give the cross section exactly the same t-dependence, though a different normalization.

Moreover, apart from minor kinematic differences, the spin-summed differential cross section for the reaction (2) has also the same t-dependence though with a different normalization, namely with $S_{\tau 1}$ in (1) replaced by $S_{\mu 1}$ in (2). For instance, from Figure 4 of [9], one deduces that in the DSM scheme, the cross section for (2) is smaller than for (1) by a factor $(S_{\mu 1}/S_{\tau 1})^2 \sim 2 \times 10^{-3}$, making it probably difficult to observe in the near future.

As in ordinary Bhabha scattering, the cross section for (1) is divergent at $t = 0$. However, this region cannot be explored by BaBar (nor presumably by Belle), the detector in which cuts out the forward region with $|t| \lesssim 5$ GeV^2 [11]. A rough estimate from Figure 6 then yields for the integrated cross section for reaction (1) over the detected angular range of BaBar a value of about 20 fb, which means about 400 events in the sample of 20 fb^{-1} already collected in last year's run assuming 100 percent detection efficiency. This is a healthy number, which seems readily detectable.

Turning next to reaction (3), the integrated cross section in the DSM can be read from Figure 5 and turns out to be about 80 fb. The reason why the cross section for (3) is larger than that for (1) despite the fact that (3) receives contribution only from the sub-dominant diagram (b) is that

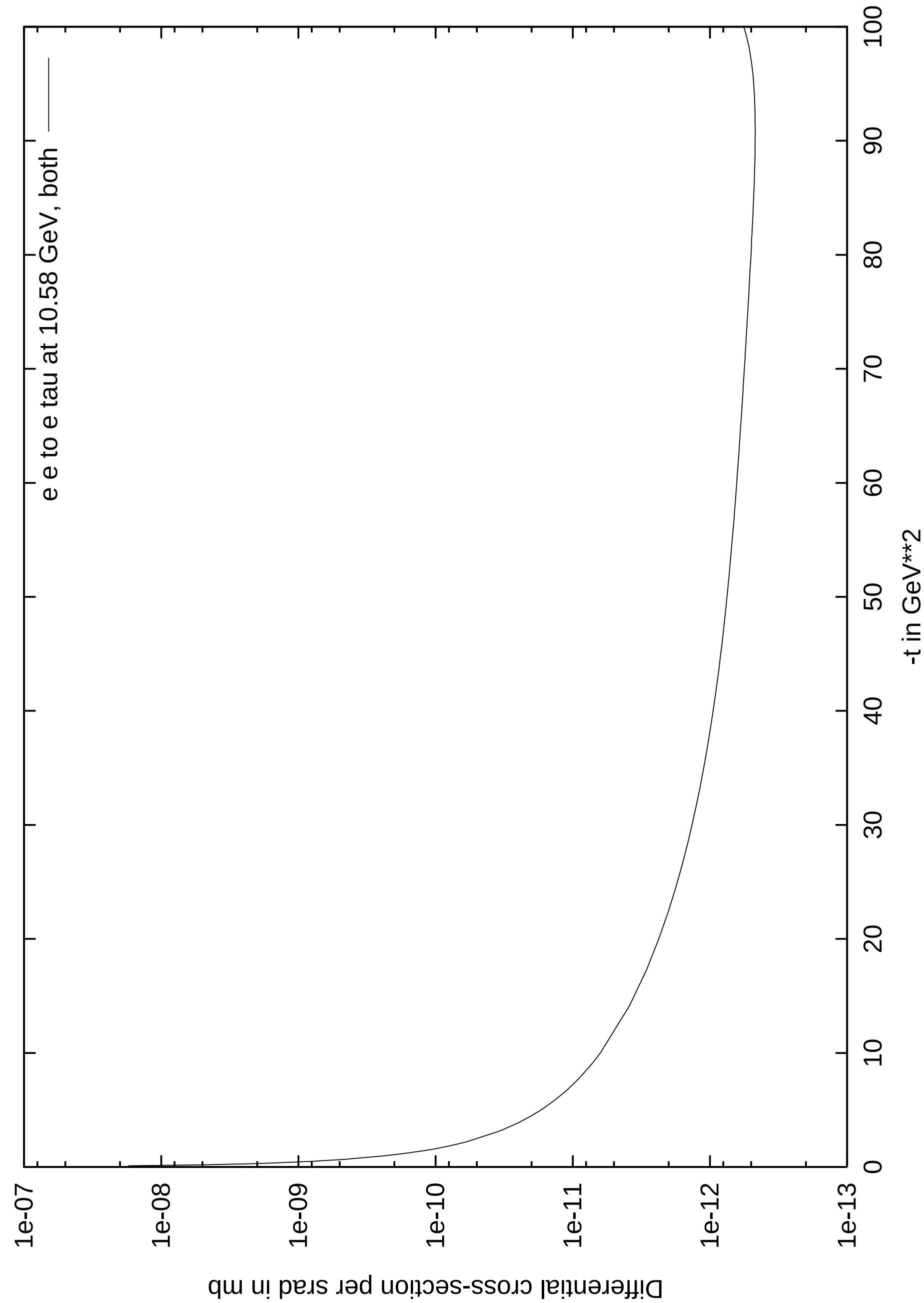


Figure 6: Spin-summed differential cross section for the reaction $e^+e^- \longrightarrow e^+\tau^-$ at $\sqrt{s} = 10.58$ GeV as calculated with rotation matrix elements $S_{\alpha 1}$ of the DSM scheme taken from ref. [5, 14].

its cross section is, according to (23), proportional to $(S_{\mu 1}S_{\tau 1})^2$ instead of $(S_{e1}S_{\tau 1})^2$ for reaction (1), and $(S_{\mu 1}/S_{e1})^2 \sim 20$ as read from Figure 4 in [9]. In practical terms, this means that as many as 1600 events may be expected from the data sample at 20 fb^{-1} already collected by BaBar, again assuming 100 percent efficiency. The spin-summed differential cross section is shown in Figure 7 and is seen to have a very different t-dependence from that of reaction (1) in Figure 6.

According to the DSM scheme, then, the cross section for both reactions (1) and (3) should be readily observable in BaBar and Belle and subjectable to detailed tests. The reason why the DSM scheme is able to make such definite predictions on the rotation matrix $S_{\alpha j}$, and hence on the normalization of transmutation cross sections, is that it is based on the assumption [8] that both the fermion mass hierarchy and the mixing of fermion states are consequences of mass matrix rotation so that the few parameters on which it depends are already fixed by fitting to the quark and lepton mass and mixing parameters. This also explains why the transmutation cross sections predicted by the DSM are so sizeable since, in order to explain the empirical masses and mixing, the mass matrix must rotate at suffient speed. Indeed, even without applying the DSM mechanism, so long as one adopts the view that it is the mass matrix rotation which gives rise to mixing and the mass hierarchy, it would be hard already to avoid transmutation effects of similar magnitudes [8].

Were we instead to take fermion mixing as a mere empirical fact as in traditional views of the Standard Model, and that it is only the mixing matrix that drives the mass matrix rotation via the renormalization group equations, then the resulting scheme would be much less predictive on transmutation and give much smaller cross sections. Of the parameters which figure in the leading-order renormalization group equations for the lepton mass matrix [3, 5], the elements of the MNS mixing matrix are still but poorly measured while the Yukawa couplings of neutrinos are largely unknown. Besides, it is unlikely that these simple equations will still apply at energies of present interest, where it is generally believed that the heavy right-handed neutrinos inherent in the see-saw mechanism [15] can be simply integrated out leaving effective couplings given only by the physical neutrino masses [16], in which case the mass matrix rotation and hence also the predicted transmutation cross sections will be very small. Even if one takes the extreme optimistic view as in the naive so-called NSM example in [5] that it is still the same equations (1.1) and (1.2) there that govern the rotation, and insert also a large value of $m_t \sim 180$GeV for the Dirac mass of the heaviest neutrino ν_3,

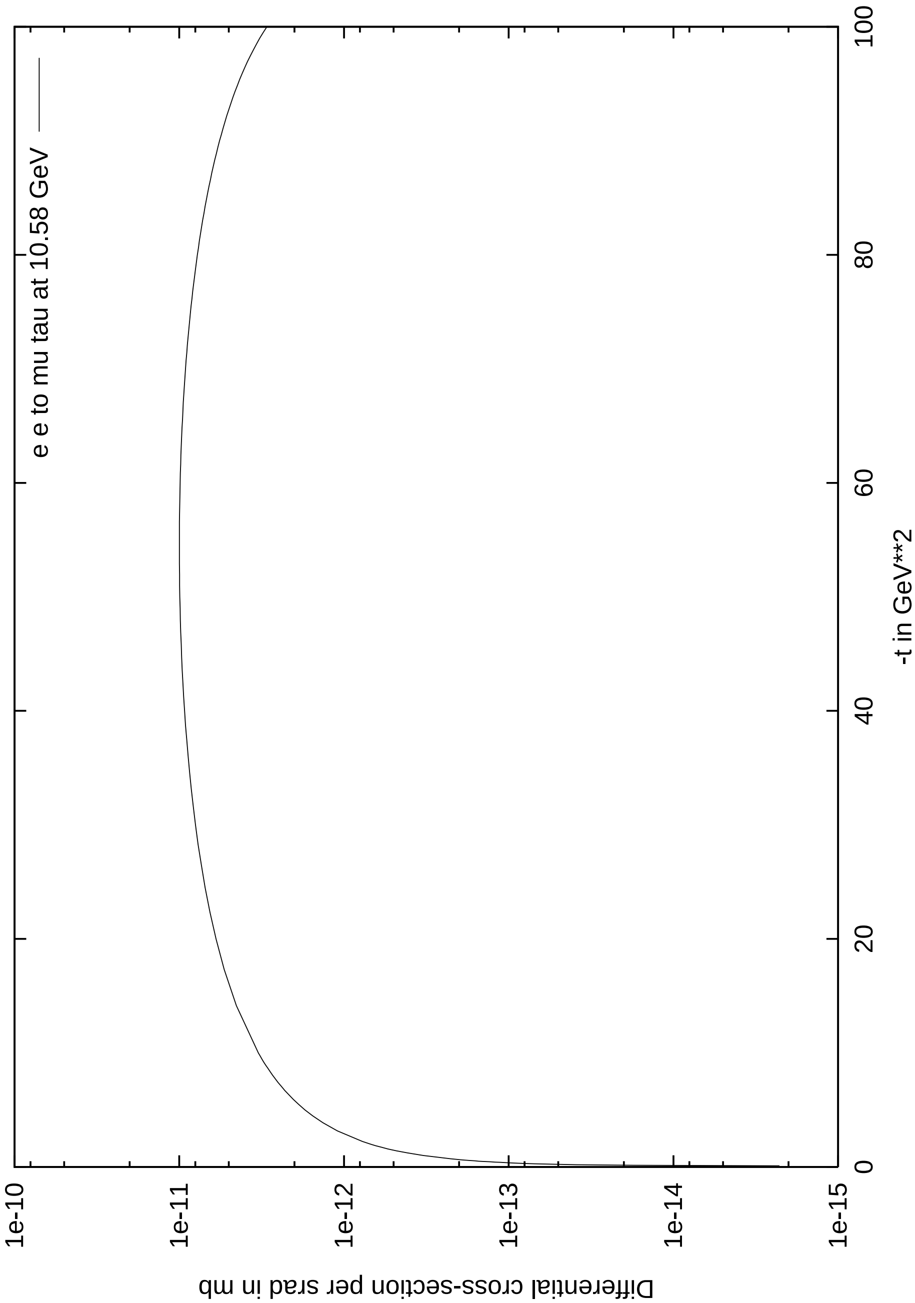


Figure 7: Spin-summed differential cross section for the reaction $e^+e^- \longrightarrow \mu^+\tau^-$ at $\sqrt{s} = 10.58$ GeV as calculated with rotation matrix elements $S_{\alpha 1}$ of the DSM scheme taken from ref. [5, 14].

one still obtains an estimate of only about 0.25 fb for the largest reaction (3), which would be difficult to observe even for ultra-sensitive experiments such as BaBar and Belle.

However, whatever rotation scheme one may choose to subscribe to, a search for the reactions (1) and (3) in the data of BaBar and Belle seems to us eminently profitable. Even a negative result putting a bound on the mass matrix rotation would be instructive. On the other hand, if the predicted effect is observed, the implications will be wide-ranging and profound, opening a new window on to the problem of generations. Both possibilities as regards the the generation puzzle and the DSM scheme have been discussed at length in [6] and need not be repeated here.

Suppose, optimistically, that the reactions (1) and (3) are indeed observed, can one be sure that they are due to transmutation and not some other lepton-violating effect? This is where the present result in e^+e^- collision wins over our previous study in vector boson decay [6]. Whereas in [6] the predictions are single numbers, namely the branching ratios of lepton flavour-violating decays, here we have the differential cross sections as functions of 2 variables, each with distinctive characteristics. For example, it is predicted that the cross section for (1) should be peaked sharply forwards as seen in Figure 6, while for (3) it should have a roughly $\sin^2\theta$ behaviour, as seen in Figure 7. And both are predicted to be spin-flip dominated, which assertion may be verifiable with the decaying τ serving as its own spin-analyser and the spin-dependent cross section calculable from the amplitudes (21) and (30) when the occasion demands. Thus, if the reactions are observed at all with any reasonable statistics, the signatures for transmutation would seem to be quite unmistakable, and given the statistics already achieved by BaBar and Belle with the promise of 10 times more in the near future, an early resolution of the question seems assured.

We have restricted the discussion in this section to only the operation energy of BaBar, Belle and Cleo, but very similar remarks apply also to the BEPC energy [17]. The predicted transmutation cross sections at BEPC, as seen in Figure 5 for reaction (3), are even larger, but this advantage is unfortunately more than offset by the lower luminosity so far achieved by the machine. For this reason, for BEPC, we think a search for transmutation in ψ decay is more immediately profitable [6]. As for LEP, according to Figure 5 for example, transmutation cross sections would have fallen much below the fb level by that energy and are thus unlikely to show up in the data collected.

6 Concluding Remarks

Since the successful advent of the Standard Model, seemingly carrying all before it, there have been few occasions where theoreticians can have the pleasure of making detailed, testable predictions on a new phenomenon free from a large number of adjustable parameters. But if our reasoning above and in [5, 9] is correct, then lepton-transmutation would seem to be just such a phenomenon. That one is able to make detailed predictions in transmutation is because it is (apart from the normalization which requires an undersanding of the rotation mechanism) basically a very simple kinematical effect, requiring for its deduction only a careful definition of the fermion flavour states in the case when the mass matrix rotates. Its simplicity, however, does not equal triviality, for if transmutation is indeed observed, then the physical implications are profound. As already explained in [6], experimental confirmation of the phenomenon not only checks the renormalization procedure in quantum field theory, but radically changes our one-time conception of flavour states as independent entities by making them merely different manifestations of the same object, presumably related by some "horizontal" symmetry.

The special beauty of the present application to e^+e^- collision is that not only explicit predictions are obtained for the differential cross sections just from the mass matrix rotation, but also that these predictions appear immediately testable with existing data. With luck, one should soon know whether transmutation does or does not occur, in either case giving valuable physical information. By far the most exciting scenario would be that the effect will be observed with a cross section at BarBar and Belle of the order predicted in Figures 6 and 5, for in that case, it seems that some headway will at last be made towards resolving the generation puzzle, as explained, for example, in [6]. For this reason, it is no exaggeration to say that an analysis along these lines is awaited with breath-bated anticipation.

We thank Gian Gopal for useful discussions on the experimental feasibility of testing the predictions of this paper.

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